Two-sensor methods for the measurement of sound intensity and acoustic properties in ducts

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A unified theoretical approach to the development of two-sensor methods is presented. It is shown that various methods developed in the last 10 years for sound intensity measurement and for the measurement of acoustic properties in ducts may be systematically derived from a general decomposition theory. In the decomposition theory, the incident and reflected wave auto and cross spectra are obtained from a set of decomposition equations using the measurement of the total acoustic pressure at two arbitrary points in a one-dimensional steady, random sound field. The application of the wave spectra to the measurement of sound intensity and acoustic properties follows directly. It is further shown that the decomposition theory predicts a set of characteristic wavenumbers at which two-sensor methods fail to yield meaningful data. Experimental data are presented showing the application of the decomposition theory to acoustic property determination, sound intensity measurement, and the estimation of sound pressure and particle velocity in a duct.

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INTRODUCTION

This article is concerned with the theoretical development of two-sensor methods for extracting information from one-dimensional sound fields in ducts. The information that is extracted is usually the sound intensity in the duct or the acoustic properties of the passive termination of the duct. Although the acoustic medium is usually air, the methods apply equally well to ducts containing other fluids such as water or oil if the tubes containing these fluids are rigid.

An early application of two-sensor methods[1] using random excitation was concerned with the decomposition of a steady-state field into incident and reflected wave spectra for the determination of acoustic impedance. Subsequently, the same experimental configuration used in Ref. 1 formed the basis of the transfer function method[2–4] for determining acoustic properties. It will be shown here that the transfer function method may be deduced from the decomposition theory in Ref. 1.

It is worth noting that the two-microphone, random excitation methods discussed in Refs. 1–4 are not alone as novel techniques to measure acoustic properties. Singh and Katra[5] developed an impulse technique in which a broadband transient pulse was generated digitally. Because the incident and reflected pulses were distinguishable in the time domain, the incident and reflected wave spectra could be computed without decomposition. Nichols[6] has described a similar pulse technique utilizing hydrophones for measuring impedance in a water-filled tube. Doige and his students developed[7,8] a broadband testing technique using transient swept sine wave excitation. Their application was the determination of the acoustic properties of mufflers. Elliott[9] has described a simple method utilizing two microphones and analog instrumentation for measuring the absorption coefficient.

Studies of the experimental errors encountered in the use of the two-microphone technique[10,11] have been reported. Bias errors occur when the spectral resolution is inadequate or when the phase or amplitude calibration is incorrect. Random errors may be large when the signal-to-noise ratio is low (e.g., when the termination is highly reflective) as evidenced by a low coherence measurement. Reference 11 contains numerous examples illustrating these errors. Recently, Chu[12] has shown that in some cases it may be necessary to include tube absorption in the two-sensor methods in order to improve the accuracy of the measured acoustic properties.

In the present article, we focus our attention on the elaboration of the decomposition theory. Our primary objective is to show that the decomposition theory provides a common base upon which several existing "new" methods are founded. It is shown that the decomposition method includes as a special case the transfer function method as well as a method proposed for measuring the sound intensity in a duct.[13] Other applications of the decomposition method are introduced including the estimation of the particle velocity spectrum and the estimation of the acoustic pressure spectrum anywhere in the duct. Finally, the decomposition theory is used to provide an interesting interpretation of the critical wavenumber problem that plagues the two-sensor methods.

I. THE DECOMPOSITION THEORY

A one-dimensional, standing-wave acoustic field, superimposed on a uniform flow, may be decomposed into incident and reflected wave spectra, $S_{AA}(f)$ and $S_{BB}(f)$, respectively, and the cross spectrum between the incident and reflected waves, $S_{AB}(f) = C_{AB}(f) + jQ_{AB}(f)$. These quantities are related to the auto and cross spectra of the total acoustic pressure at two arbitrary points in the field (see Fig. 1) through the following system of equations (dropping the frequency f for brevity)[1]:

$$[A][S_{AA}\ S_{BB}\ C_{AB}\ Q_{AB}]^T = [S_{11}\ S_{22}\ C_{12}\ Q_{12}]^T. \quad (1)$$

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In the above equations, $S_{11}(f)$ and $S_{22}(f)$ are the auto spectra of the total acoustic pressure at arbitrary points x_1 and x_2, respectively, and $S_{12}(f) = C_{12}(f) + jQ_{12}(f)$ is the cross spectrum between the total acoustic pressure at these points. The vectors on the left and right sides of Eq. (1) are referred to as the wave spectra matrix and the total pressure spectra matrix, respectively. The elements of the A matrix are

$$\begin{aligned}
&a_{11} = a_{12} = a_{21} = a_{22} = 1 ,\\
&a_{13} = 2\cos(k_i + k_r)x_1 ,\\
&a_{14} = 2\sin(k_i + k_r)x_1 ,\\
&a_{23} = 2\cos(k_i + k_r)x_2 ,\\
&a_{24} = 2\sin(k_i + k_r)x_2 ,\\
&a_{31} = \cos k_i(x_1 - x_2) ,\\
&a_{32} = \cos k_r(x_1 - x_2) . \qquad (2)\\
&a_{33} = \cos(k_r x_1 + k_i x_2) + \cos(k_i x_1 + k_r x_2) ,\\
&a_{34} = \sin(k_r x_1 + k_i x_2) + \sin(k_i x_1 + k_r x_2) ,\\
&a_{41} = \sin k_i(x_1 - x_2) ,\\
&a_{42} = -\sin k_r(x_1 - x_2) ,\\
&a_{43} = \sin(k_i x_1 + k_r x_2) - \sin(k_r x_1 + k_i x_2) ,\\
&a_{44} = \cos(k_r x_1 + k_i x_2) - \cos(k_i x_1 + k_r x_2) ,
\end{aligned}$$

In the above equations, $k_i = k/(1+M)$, $k_r = k/(1-M)$, and $k = 2\pi f/c$, where c is the speed of sound and M is the uniform flow Mach number. It should be noted that, according to the sign convention adopted in Fig. 1, x_1 and x_2 are negative and $x_1 - x_2$ is positive.

In Ref. 1, the wave spectra S_{AA}, S_{BB}, C_{AB}, and Q_{AB} were determined for a specific measurement by solving Eq. (1) numerically. Although obtaining the numerical solution to Eq. (1) is efficient and trivial, even on a personal computer, there is ample motivation to obtain the analytical solution. Equation (1) may be solved to yield the wave spectra matrix

$$[S_{AA}\ S_{BB}\ C_{AB}\ Q_{AB}]^T = \delta[B][S_{11}\ S_{22}\ C_{12}\ Q_{12}]^T , \quad (3)$$

where, after much algebra, $\delta[B] = [A]^{-1}$ has the simple result

$$\delta[B] = \delta\begin{bmatrix} 1 & 1 & -2a_{32} & -2a_{42} \\ 1 & 1 & -2a_{31} & -2a_{41} \\ -a_{23}/2 & -a_{13}/2 & a_{33} & a_{43} \\ -a_{24}/2 & -a_{14}/2 & a_{34} & a_{44} \end{bmatrix} \quad (4)$$

and $\delta = 1/4\sin^2[(k_i + k_r)(x_1 - x_2)/2]$.

The significance of Eqs. (3) and (4) is that now we have formulas for the direct decomposition of a steady random sound field into its component wave spectra. These wave spectra are the ingredients of the sound field in a tube or duct. Once the wave spectra are determined from Eq. (3), all other essential information about the sound field and the tube may be determined. Further, Eq. (3) may be used to show the correspondence between the decomposition theory and other methods[3,4,13] and to develop other applications.

An important application of the wave spectra is the determination of the acoustic properties at the passive end of a tube or other acoustic waveguide containing a one-dimensional random sound field. The normalized impedance $Z_0 = R_0 + jX_0$ and the power reflection coefficient α at the passive end of the tube are related to the wave spectra by[1]

$$\begin{aligned}
R_0(f) &= [S_{AA}(f) - S_{BB}(f)]/\\
&\quad [S_{AA}(f) + S_{BB}(f) - 2C_{AB}(f)] , \qquad (5)\\
X_0(f) &= -2Q_{AB}(f)/[S_{AA}(f) + S_{BB}(f) - 2C_{AB}(f)] ,
\end{aligned}$$

and

$$\alpha_0(f) = S_{BB}(f)/S_{AA}(f) . \quad (6)$$

Equations (5) and (6) will be discussed further in Sec. III.

II. SOUND INTENSITY IN A DUCT WITH FLOW

The magnitude of the sound intensity I_i of the incident wave spectrum is

$$I_i = (1+M)^2 S_{AA}/\rho c \quad (7a)$$

and the magnitude of the sound intensity I_r of the reflected wave spectrum is

$$I_r = (1-M)^2 S_{BB}/\rho c , \quad (7b)$$

where ρc is the characteristic impedance of the medium. In Ref. 13, the authors discuss a transfer function technique for measuring sound intensity in a duct. The sound intensity formulation in Ref. 13 may be deduced more directly from the decomposition theory, as follows. Using the first two relationships in Eq. (3), we may rewrite the above expressions as

$$\begin{aligned}
I_i = \delta(1+M^2)[&S_{11} + S_{22} - 2C_{12}\cos k_r(x_1 - x_2)\\
&+ 2Q_{12}\sin k_r(x_1 - x_2)]/\rho c \qquad (8)
\end{aligned}$$

and

$$\begin{aligned}
I_r = \delta(1-M)^2[&S_{11} + S_{22} - 2C_{12}\cos k_i(x_1 - x_2)\\
&- 2Q_{12}\sin k_i(x_1 - x_2)]/\rho c . \qquad (9)
\end{aligned}$$

The magnitude of the net sound intensity $I = I_i - I_r$ from Eqs. (8) and (9) is

$$\begin{aligned}
I = (\delta S_{11}/\rho c)\{&(1+M)^2|(S_{12}/S_{11})\\
&- \exp[-jk_r(x_1 - x_2)]|^2\\
&- (1-M)^2|(S_{12}/S_{11}) - \exp[jk_i(x_1 - x_2)]|^2\} , \qquad (10)
\end{aligned}$$

where the identity $S_{22} = |S_{12}|^2/S_{11}$ has been used. For the case where $M = 0$, Eq. (10) reduces to the simple result

$$I = Q_{12}/\rho c \sin k(x_1 - x_2) . \quad (11)$$

Equation (11) may be obtained more directly from Eq. (1) as follows. The fourth relationship in Eq. (1) is

$$a_{41}S_{AA} + a_{42}S_{BB} + a_{43}C_{AB} + a_{44}Q_{AB} = Q_{12} .$$

When $M = 0$, $a_{41} = \sin k(x_1 - x_2) = -a_{42}$ and $a_{43} = a_{44} = 0$, from Eq. (2), so that the above relationship becomes

$$S_{AA} - S_{BB} = Q_{12}/\sin k(x_1 - x_2) . \quad (12)$$

The net sound intensity is found by dividing Eq. (12) by ρc; this result is the same as Eq. (11).

If the transfer function $H_{12} = S_{12}/S_{11}$ is substituted into Eqs. (10) and (11) and if a reverse designation of the measurement locations in Ref. 13 is accounted for, the results are identical to equations presented in Ref. 13, except for the sign error pointed out recently.[14] Thus the net sound intensi-

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ty, Eq. (10), is a special result obtained from the general decomposition theory in Ref. 1; and, if $M=0$, the net sound intensity, Eq. (11), is found without decomposition from Eq. (1).

III. MEASUREMENT OF ACOUSTIC PROPERTIES

The two-microphone, random excitation technique, Fig. 1, may be used for the determination of the normal acoustic properties in a tube. In this technique, the tube is driven by random sound, and the total acoustic pressure is measured at two points in the tube. From these measured pressure time records, estimates of the quantities $S_{11}(f)$, $S_{22}(f)$, and $S_{12}(f)$ are obtained. With certain precautions to reduce random and bias errors, these estimates may be used to replace the true quantities in Eq. (3). After the wave spectra are determined from Eq. (3), the acoustic properties of the passive end of the tube may be found from Eqs. (5) and (6).

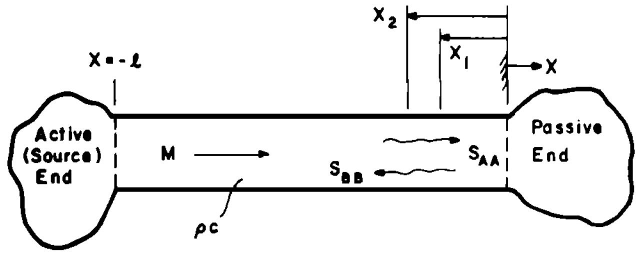

FIG. 1. Duct with arbitrary measurement points x_1 and x_2.

In Refs. 3 and 4, the authors describe an identical measurement scheme except that the acoustic properties are calculated directly from a transfer function $H_{12}(f)$, defined previously. In what follows, we show that the transfer function approach is a special result of the general decomposition theory. For simplicity, consider the no-flow case; when $M=0$, Eq. (3) becomes

$$S_{AA} = [S_{11} + S_{22} - 2C_{12}\cos k(x_1 - x_2) + 2Q_{12}\sin k(x_1 - x_2)]/4\sin^2 k(x_1 - x_2)\ ,$$
$$S_{BB} = [S_{11} + S_{22} - 2C_{12}\cos k(x_1 - x_2) - 2Q_{12}\sin k(x_1 - x_2)]/4\sin^2 k(x_1 - x_2)\ ,$$
$$C_{AB} = [-S_{11}\cos 2kx_2 - S_{22}\cos 2kx_1 + 2C_{12}\cos k(x_1 + x_2)]/4\sin^2 k(x_1 - x_2)\ ,$$
$$Q_{AB} = [-S_{11}\sin 2kx_2 - S_{22}\sin 2kx_1 + 2C_{12}\sin k(x_1 + x_2)]/4\sin^2 k(x_1 - x_2)\ .$$

Substituting the above expressions into Eqs. (5), we obtain

$$X_0 = \frac{S_{22}\sin kx_1\cos kx_1 - C_{12}\sin k(x_1 + x_2) + S_{11}\sin kx_2\cos kx_2}{S_{11}\cos^2 dx_2 + S_{22}\cos^2 kx_1 - 2C_{12}\cos kx_1\cos kx_2}$$

and

$$R_0 = \frac{Q_{12}\sin k(x_1 - x_2)}{S_{11}\cos^2 kx_2 + S_{22}\cos^2 kx_1 - C_{12}\cos kx_1\cos kx_2}\ .$$

The above equations may be rewritten in terms of the transfer function H_{12} by noting that $C_{12}/S_{11} = \mathrm{Re}(H_{12})$, $Q_{12}/S_{11} = \mathrm{Im}(H_{12})$, and $S_{22}/S_{11} = |H_{12}|^2$:

$$X_0 = \frac{|H_{12}|^2\sin kx_1\cos kx_1 - \mathrm{Re}(H_{12})\sin k(x_1 + x_2) + \sin kx_2\cos kx_2}{\cos^2 kx_2 + |H_{12}|^2\cos^2 kx_1 - 2\,\mathrm{Re}(H_{12})\cos kx_1\cos kx_2} \qquad (13)$$

and

$$R_0 = \frac{\mathrm{Im}(H_{12})\sin k(x_1 - x_2)}{\cos^2 kx_2 + |H_{12}|^2\cos^2 kx_1 - 2\,\mathrm{Re}(H_{12})\cos kx_1\cos kx_2}\ . \qquad (14)$$

Equations (13) and (14) are the same as those presented in Refs. 3 and 4 except for a sign change owing to a reverse designation of the microphone numbers.

It is now apparent that the transfer function method of determining acoustic properties may be shown to be a special result of the decomposition theory. However, this has not been demonstrated before, and apparently was not known to the authors of Refs. 3 and 4. As a practical matter, the decomposition method and the transfer function method give identical results when applied to experimental data. There are instances, however, when the decomposition method is preferred, as it provides more information than the transfer function method. Because many acoustic materials and systems of practical interest exhibit nonlinear behavior, the measured impedance must be qualified with some indication of the sound-pressure amplitude at which the test was made. The auto spectrum of the incident wave S_{AA} or the total sound pressure at the face of the sample under test may be used for this purpose. Because the transfer function method does not utilize all the data available (i.e., only H_{12} is measured), it is not possible to determine S_{AA}. In principle, it should be possible to design a test procedure using the decomposition method to provide a specified sound-pressure spectrum to the sample under test.

In deriving Eqs. (13) and (14), the common factor $\sin^2 k(x_1 - x_2)$ was cancelled from the numerator and denominator in both X_0 and R_0. This, of course, can only be done when the common factor is nonzero. This leads to a set

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of characteristic wavenumbers (to be derived subsequently) at which X_0 and R_0 may not be determined.

IV. OTHER APPLICATIONS OF THE DECOMPOSITION THEORY

Once the decomposition in Eq. (3) has been accomplished, we may use the wave spectra to estimate the total acoustic pressure anywhere in the duct. For example, the spectrum of the total acoustic pressure S_{pp} at any point x is

$$S_{pp} = S_{AA} + S_{BB} + 2[C_{AB}\cos(k_i + k_r)x + Q_{AB}\sin(k_i + k_r)x], \quad (15)$$

which was obtained from the first relation in Eq. (1) by letting $x_1 = x$. Equation (15) is useful for determining the total acoustic pressure at the face of a sample which has an amplitude-dependent impedance. The spectrum of the total acoustic particle velocity at any point x is[1]

$$S_{uu} = (1/\rho c)^2\{S_{AA} + S_{BB} - 2[C_{AB}\cos(k_i + k_r)x - Q_{AB}\sin(k_i + k_r)x]\}. \quad (16)$$

V. CHARACTERISTIC WAVENUMBERS

In Refs. 3 and 4, the authors note that there is an upper-frequency bound for the measurement of acoustic properties that is determined by the lesser of: (1) the cutoff frequency of the tube or (2) a frequency corresponding to a microphone separation distance of one-half wavelength. However, with respect to (1), Waterhouse[15] has pointed out that the plane-wave mode can be extended to frequencies above cutoff. Likewise, condition (2) is not an upper bound because data can be obtained at frequencies above the one-half wavelength microphone spacing.[1] However, how to obtain data at frequencies in the neighborhood of the frequency corresponding to (2) without using a third microphone measurement position remains an open question.

The microphone separation distance $L = x_1 - x_2$ establishes a set of characteristic wavenumbers at which the decomposition [Eq. (3)] is indeterminate. From Eqs. (1) and (2)

$$\text{DET}[A] = -16\sin^4(k_i + k_r)(x_1 - x_2)/2. \quad (17)$$

This relation defines a set of characteristic wavenumbers k_n when $\text{DET}[A] = 0$, at which the decomposition is singular. That is,

$$k_n = [n\pi(1 - M^2)]/L, \quad n = 0,1,2,\ldots. \quad (18)$$

At all frequencies except those defined by Eq. (18), the decomposition will be unique. Note that, when $k = k_n$, it does not appear possible to evaluate either the sound intensity in the duct [Eq. (10) or (11)] or the acoustic properties at the passive end [Eqs. (13) and (14)].

In the neighborhood of k_n, small measurement errors will cause large errors in the decomposition, in the evaluation of sound intensity, and in the evaluation of acoustic properties. This error may be reduced by reducing the random errors on the estimate of S_{11}, S_{22}, and S_{12} in the neighborhood of the critical wavenumbers.[10] The condition number of $[A]$ may be used to indicate the relative magnitude of

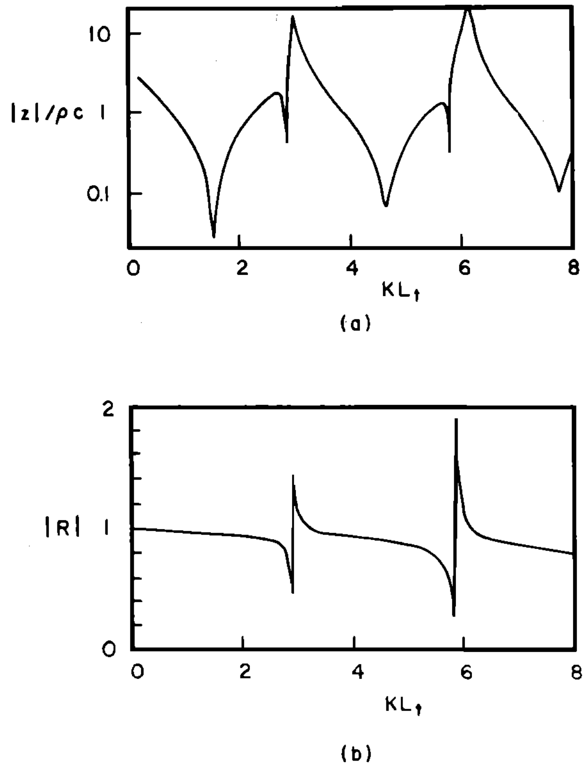

FIG. 2. Measured impedance and reflection coefficient of a tube of length L_t closed at one end.

these errors.[16] Thus measured data in the neighborhood of k_n can be eliminated as inaccurate.

The extent of the ill-conditioning problem at frequencies in the neighborhood of k_n may be seen from Fig. 2. Figure 2 shows the magnitude of the impedance and the reflection coefficient of a tube of length L_t closed at one end and attached to a waveguide filled with oil under high pressure. The data in Fig. 2 were obtained using the two-sensor method and piezoelectric pressure transducers separated by approximately 1 m. The theoretical impedance of a tube of length L_t is $|\cot kL_t|$, and the theoretical reflection coefficient is $|R| = 1$. Resonance in the tube occurs at frequencies corresponding to $kL_t = m\pi/2$, $m = 1,3,5,\ldots$; these frequencies are clearly visible in Fig. 2(a). However, it may also be seen in Fig. 2(a) that the ill conditioning in the neighborhood of the k_n mimics resonances at frequencies near $kL_t = 3$ and 6, making the interpretation of impedance difficult. From Fig. 2(b), it may be seen that the region of ill conditioning extends significantly on either side of the k_n.

The frequencies at which the denominator of Eqs. (13) and (14) is zero are the same as the characteristic frequencies predicted by Eq. (18). In the neighborhood of these frequencies, the transfer function technique fails to yield meaningful data. However, that these frequencies are due to the singular behavior of the $[A]$ matrix is hidden by the transfer function method. The (physical) formulation of the impedance shown in Eq. (5) does not have a singularity, in general, and the true cause of the error in the neighborhood of the characteristic frequencies in the transfer function method is due to the singular behavior of the decomposition in Eq. (3). Equation (5) shows that, if it is possible to deter-

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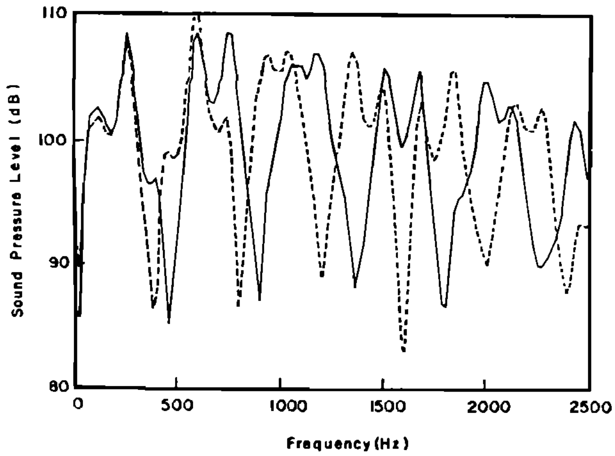

FIG. 3. Sound-pressure spectra measured in the duct: solid line, S_{11}; broken line, S_{22}.

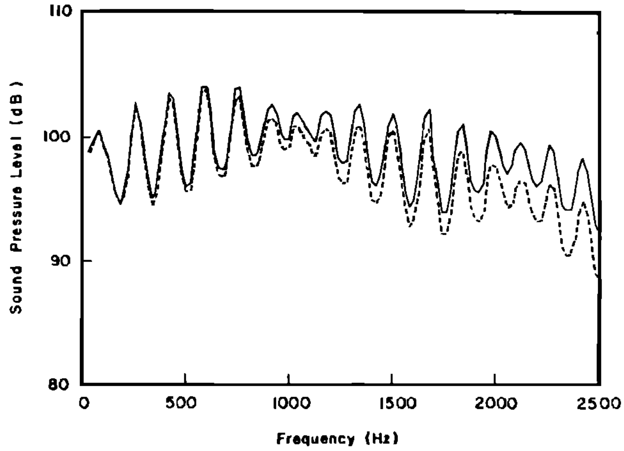

FIG. 5. Decomposed wave spectra in the duct: solid line, S_{AA}; broken line, S_{BB}.

mine the wave spectra accurately, it is always possible to calculate the impedance.

The characteristic frequencies in Eq. (18) are the same frequencies at which the transfer function method for sound intensity, Eq. (10), fails. The failure of the transfer function method at these frequencies was not discussed in Ref. 13.

VI. EXPERIMENTAL RESULTS

In this section, experimental results are presented illustrating the application of the decomposition method to an air-filled circular duct, similar to Fig. 1, with $M = 0$. The source of sound was a loudspeaker, driven by bandlimited white noise. The duct was terminated at the passive end by an open tube approximately 152 mm long. The first microphone was located 216 mm from the entrance to the open tube, and the spacing between the microphones was 50.8 mm. The pressure signals were digitized and analyzed using a two-channel spectrum analyzer that was controlled by an IBM PC/XT computer.

Figures 3 and 4 show the measured auto and cross spectra of the total sound pressure (S_{11}, S_{22}, and S_{12}) at the two microphone locations in the tube. The $n = 1$ characteristic frequency for a microphone spacing of 50.8 mm is approximately 3400 Hz, as determined from Eq. (18). The data above 2500 Hz in Figs. 3 and 4 and in subsequent figures contain possible errors due to the ill conditioning discussed in Sec. V and have not been shown. The large peaks in the spectra in Figs. 3 and 4 are due to the resonances of the duct and the attached open tube. The minima in Figs. 3 and 4 occur at frequencies in which a pressure minimum coincides with one of the microphones. In the frequency range where a pressure minimum coincides with a microphone, there is a corresponding phase shift of approximately 180°, Fig. 4, as expected for plane standing waves in a duct.

In Figs. 5 and 6, the wave spectra S_{AA}, S_{BB}, and S_{AB} are shown. The wave spectra were obtained by using the measured spectra in Figs. 3 and 4 with Eq. (3). Upon examination of Fig. 5, it may be seen that the spectrum of the waves incident on the termination (i.e., S_{AA}) and the intensity of the incident waves, Eq. (7a), are strongly influenced by the resonances of the tube. This effect, which is well known and

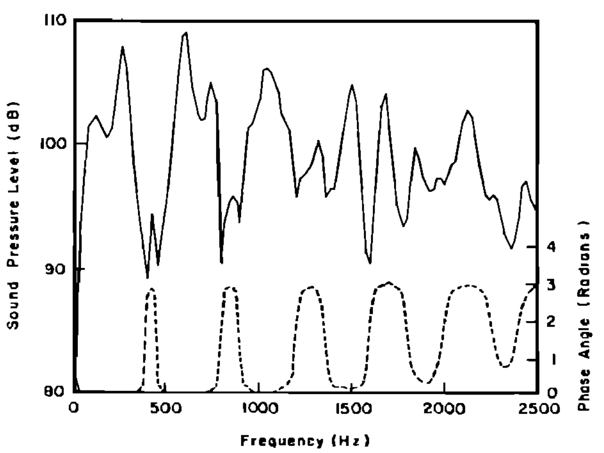

FIG. 4. Cross spectrum measured in the duct: solid line, magnitude of S_{12}; broken line, phase of S_{12}.

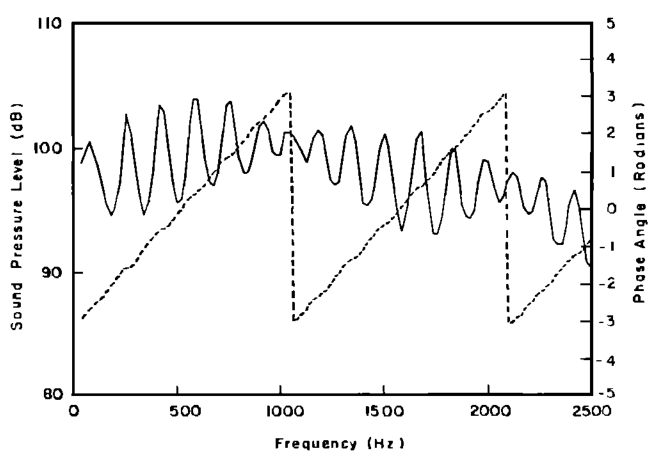

FIG. 6. Decomposed wave spectra in the duct: solid line, magnitude of S_{AB}; broken line, phase of S_{AB}.

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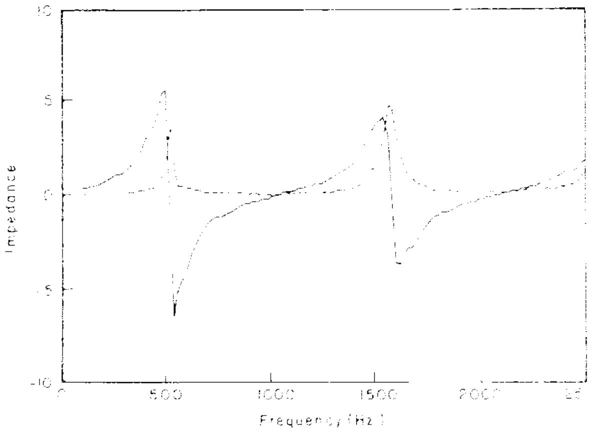

FIG. 7. Impedance of a 152-mm-long open tube: solid line, reactance; broken line, resistance.

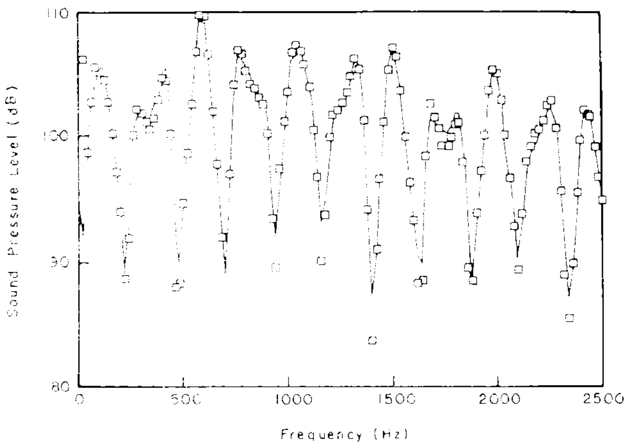

FIG. 9. Measured (solid line) and computed (open square) sound-pressure level in the duct at a point 572 mm from the entrance of the tube.

which is important when testing nonlinear materials and systems, can be quantified with data such as those presented in Fig. 5.

The data in Figs. 5 and 6 were used along with Eq. (5) to determine the impedance of the open tube attached to the end of the duct. The impedance of the open tube is shown in Fig. 7. An approximate theoretical model that neglects the radiation impedance shows that the first two natural frequencies of the open tube occur at 1125 and 2250 Hz, respectively. These results agree well with the data in Fig. 7.

Figure 8 shows the net sound intensity in the duct obtained by using the data in Figs. 5 and 6 in Eq. (10) with $M = 0$ [or, alternately, in Eq. (11)]. As there are no losses in the tube, the net sound intensity in Fig. 8 is the same sound intensity at the open end of the tube. Characteristic of the open tube, much more energy passes out of the tube at high frequency than at low frequency.

Figure 9 shows the total sound pressure measured at a point 572 mm from the passive end of the duct. Also shown in Fig. 9 is the sound pressure predicted using the wave spectra in Figs. 5 and 6 with Eq. (15). The agreement between the measured and computed sound-pressure spectra is quite good, even though tube absorption was not included.

Figure 10 shows another example of the use of the wave spectra to calculate acoustic variables in the duct. In Fig. 10, the calculated sound-pressure spectrum and particle velocity spectrum at the duct termination (i.e., at the entrance of the tube) are shown. These spectra were computed using the wave spectra in Figs. 5 and 6 and Eqs. (15) and (16). The particle velocity was normalized by a reference particle velocity $u_0 = p_0/\rho c$, where $p_0 = 0.00002$ Pa is the reference acoustic pressure in air. From the data in Fig. 10, it is quite easy to see the approximate location of the resonance and antiresonance frequencies of the tube (see Fig. 7) as those where the sound-pressure spectrum and particle velocity spectrum, respectively, have minima.

In the experiment reported above, the effects of random and bias errors were carefully monitored. Data were rejected if the coherence was less than 0.95; thus there are no results below 100 Hz in Fig. 7, for example. Bias errors due to inadequate spectral resolution were avoided by increasing the spectral resolution repeatedly until consistent spectra were obtained.

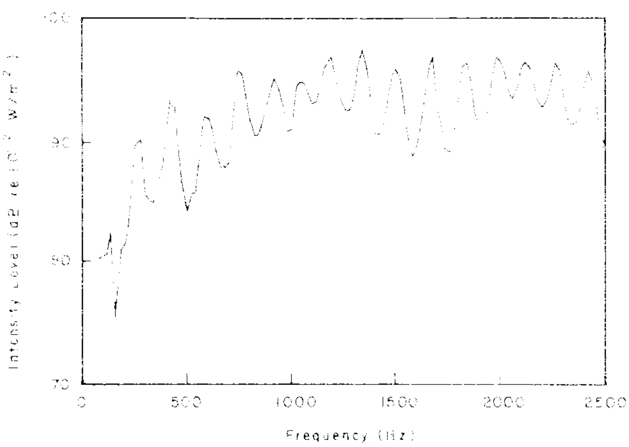

FIG. 8. Net sound intensity in the duct.

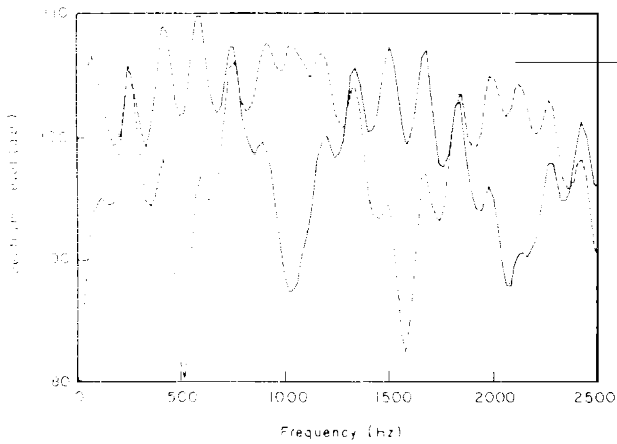

FIG. 10. Sound-pressure level and particle velocity level computed at the entrance of the tube: solid line, sound-pressure level; broken line, particle velocity level (re: 482 μm/s).

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VII. SUMMARY

Several fundamental aspects of two-sensor methods have been discussed. It has been shown that two-sensor methods for various applications can be derived in a more general way using the decomposition theory. In particular, the transfer function techniques[3,4,13] have been shown to be special cases of the more general decomposition theory. It has also been shown that the sensor separation problem is a manifestation of the familiar ill conditioning that accompanies the solution of a system of near-singular equations when finite precision data are used. Further applications of the decomposition method, including the estimation of the total acoustic pressure and particle velocity anywhere in a waveguide, have been shown to be feasible.

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